



VIBRATION OF A FLEXIBLE PIPE CONVEYING VISCOUS PULSATING FLUID FLOW

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The non-linear equations of motion of a flexible pipe conveying unsteadily flowing fluid are derived from the continuity and momentum equations of unsteady flow. These partial differential equations are fully coupled through equilibrium of contact forces, the normal compatibility of velocity at the fluid–pipe interfaces, and the conservation of mass and momentum of the transient fluid. Poisson coupling between the pipe wall and fluid is also incorporated in the model. A combination of the finite difference method and the method of characteristics is employed to extract displacements, hydrodynamic pressure and flow velocities from the equations. A numerical example of a pipeline conveying fluid with a pulsating flow is given and discussed.

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1. INTRODUCTION

Unsteady flow-induced vibration, which often occurs due to pump and valve operations in pipeline systems and even in human circulation, is of concern to the hydropower and petroleum industries and biomedical engineering research. A recent and comprehensive literature review has already been published [1], where the complex problem of pipes conveying fluid with a pulsating flow is outlined. Extensive investigations on this subject have been carried out [2–5]. In these studies, the stability and resonance of pipes conveying fluid with an unsteady flow which has a time-dependent harmonic component superposed on a steady flow are examined. The effects of Poisson coupling and the pressure wave on the motion of the pipe–fluid system have not, however, been considered. Such effects are also not incorporated in a more general set of equations which has been proposed by Semler *et al.* [6]. This neglect of Poisson coupling and waterhammer wave effects is appropriate for a pipeline conveying a pulsating fluid flow with a small pulsating frequency. However, in the case of fluid-conveying pipelines with a larger pulsating frequency, the effects of the Poisson coupling and the pressure wave should be incorporated in the problem.

Even though numerous recent attempts have been made to couple the motion of the pipe and fluid accurately, with the inclusion of the Poisson coupling and pressure wave effects [7–9], there remains a need to investigate all mechanisms for fluid–structure interaction. Wiggert *et al.* [10] developed an approach to predict

liquid pressure and pipe dynamic response based on the method of characteristics, which was applied to the Timoshenko beam theory to examine the axially coupled behaviour of liquid-filled pipes. This method, however, has its limitations, including the necessity of using a very small time step due to the extremely high wave speed within the pipe wall. Also, the equations of the waterhammer effect are not fully coupled with the vibration of the fluid-conveying pipe in both the lateral and longitudinal directions in this model. Recently, a more general set of equations governing the motion of the pipe and fluid has been obtained by Lee *et al.* [11]. However, the partial differential equations derived are not fully coupled, and the mechanism of the fluid–structure interaction is only partially illustrated. Even for a higher flow pulsation frequency, the effect of the pressure wave on the dynamic response was not considered in their example. Later, Lee *et al.* [12] further improved their previous model by including a circumferential strain effect, caused by the internal fluid pressure. Thus, the effects of Poisson and friction coupling mechanisms are fully considered, but radial shell vibration of pipes was not considered in their finite element formulation.

This paper presents the authors' attempt to formulate an improved and generalized model which includes the effects of radial shell vibration of pipes and initial axial tensions within the pipes besides both the Poisson and friction coupling mechanisms. In our model the influence of vibration on hydrodynamic pressure and flow velocity is accounted for, and *vice versa*. Initial axial tension and external excitation are also considered in our model so that it may be applied to the analysis of initially stretched tubes conveying fluid with a pulsating flow. The pressure pulsation and dynamic response are investigated and an example is given to illustrate an application of our model.

2. MODEL FORMULATION

Consider one-dimensional pulsating flow of a Newtonian viscous incompressible fluid in a circular flexible pipe with length L , cross-sectional area A_t , mass per unit length m_t , initial axial tension T_0 , conveying a fluid of mass m_f per unit length with axial velocity U , and pressure p , which varies with time and space. This fluid-conveying pipe, which is simply supported at both ends, is subjected to an external excitation. For generality, the pipeline is inclined at an angle of ϕ degrees. The pipe wall material is treated as a linear, homogeneous and isotropic elastic medium.

Let O be the origin at the centreline of the pipe at the point of support and let the local co-ordinate x be in the pipe neutral axis at equilibrium and y normal to it. The external excitation displacement $w_0(t)$ will be identical at any point within the whole system. The pipeline experiences a transverse displacement w and an axial displacement u from its equilibrium position at time $t = 0$. The pipe wall also experiences radial constriction and expansion due to the unsteady flow. The mechanism of the fluid–structure interaction is represented by the forces exerted by the fluid flowing on the pipe wall, and *vice versa*. On the fluid–pipe interfaces the contact force resultant is resolved into normal and tangential forces f_n and f_t (the friction force) per unit length acting on the pipe walls and *vice versa*. These forces are indicated in Figure 1.

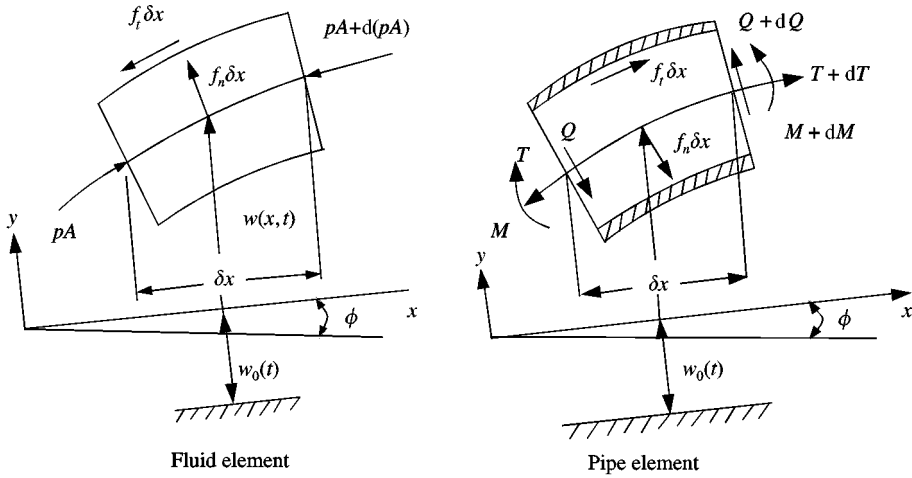


Figure 1. Fluid and pipe elements together with an indication of forces.

Upon taking a small fluid and pipe element with length δx , force balances on a liquid element, and pipe element in the x and y directions lead to, respectively, the lateral and longitudinal equations of motion of a fluid element:

$$-f_t \delta x \cos \theta - f_n \delta x \sin \theta - (pA \cos \theta)' \delta x - m_f g \delta x \sin \phi = m_f \delta x D^2(x + u)/Dt^2, \quad (1)$$

$$f_n \delta x \cos \theta - f_t \delta x \sin \theta - (pA \sin \theta)' \delta x - m_f g \delta x \cos \phi = m_f \delta x D^2(w + w_0)/Dt^2, \quad (2)$$

where p is the hydrodynamic pressure, A is the cross-sectional area of the fluid, t is the time, w_0 is the distance between the x co-ordinate and the datum line, g is the acceleration due to gravity, θ is the angle between the pipe element position and the x -axis, D/Dt is the material derivative, the prime ($'$) denotes a derivative with respect to x , and

$$\sin \theta = w'(1 - u' - \frac{1}{2}(w')^2), \quad \cos \theta = 1 - \frac{1}{2}(w')^2, \quad (3)$$

the lateral and longitudinal equations of motion of a pipe element:

$$(T \cos \theta)' \delta x - (Q \sin \theta)' \delta x + f_t \delta x \cos \theta + f_n \delta x \sin \theta - m_t g \delta x \sin \phi = m_t \delta x \ddot{u}, \quad (4)$$

$$(T \sin \theta)' \delta x + (Q \cos \theta)' \delta x - f_n \delta x \cos \theta + f_t \delta x \sin \theta - c \delta x \dot{w} - m_t g \delta x \cos \phi = m_t \delta x (\ddot{w} + \ddot{w}_0), \quad (5)$$

where T is the axial tension, Q is the resultant shear force, c is the coefficient of structural damping of the pipe, and $\dot{}$ denotes a partial derivative with respect to time t .

Strain comprises two components, namely a steady state strain due to an external tension resultant, T_0 , and an oscillatory strain, ε , due to the pipe vibration.

The axial tension, T , and the axial strain, ε , can be expressed, respectively, as

$$T = T_0 + EA_t\varepsilon, \quad \varepsilon = (u' + \frac{1}{2}(w')^2). \quad (6a, b)$$

where E is Young's modulus of the pipe. By substituting the term $(f_n \cos \theta - f_t \sin \theta)$ from equation (2) into equation (5), then using equations (3) and (6), the following equation, after some manipulation, can be obtained:

$$\begin{aligned} m\ddot{w} + c\dot{w} + 2m_f U\dot{w}' + m_f \dot{U}w' + m_f UU'w' + (pA)'w' + m_f U^2w'' - T_0w'' + (pA)w'' \\ + EIw'''' - (pA)(u''w' + u'w'' + \frac{3}{2}w'^2w'') + (T_0 - EA_t)(u''w' + u'w'' + \frac{3}{2}w'^2w'') \\ - mg \cos \phi - EI(3u''w'' + 4u''w'' + 2u'w'''' + w'u'''' + 2w'^2w'''' \\ + 8w'w''w'' + 2w''^2) + m\ddot{w}_0 = 0, \end{aligned} \quad (7)$$

where I is the inertia moment of the pipe and $m = m_f + m_t$.

By eliminating the term $(-f_t \cos \theta - f_n \sin \theta)$ from equations (1) and (4), then using equations (3) and (6), the following equation is obtained:

$$\begin{aligned} m\ddot{u} + 2m_f U\dot{u}' + m_f \dot{U}u' + m_f U^2u'' - EA_t u'' + (T_0 - EA_t)w'w'' \\ - EI(w''''w' + w''w''') - (pA)w'w'' + m_f \dot{U} + m_f UU' + (pA)' - mg \sin \phi = 0. \end{aligned} \quad (8)$$

Note that when the terms $(pA)'$, U' , \ddot{w}_0 and c are neglected, equations (7) and (8) become identical to the equations of motion derived by Semler *et al.* [6].

Upon considering the conservation of mass [13], the continuity equation of a fluid with unsteady flow is found to be

$$\rho_f \frac{DA}{Dt} + A \frac{D\rho_f}{Dt} + \rho_f A \frac{\partial U}{\partial x} = 0. \quad (9)$$

The fluid bulk modulus of elasticity and the time rate of change of the cross-sectional area of a control volume are given, respectively, by [13]

$$K = \rho_f \frac{Dp/Dt}{D\rho_f/Dt}, \quad \frac{1}{A} \frac{DA}{Dt} = \frac{D_p}{Eh} \left(\frac{Dp}{Dt} - \frac{\nu}{2A} \frac{DT}{Dt} \right), \quad (10a, b)$$

where K is the fluid bulk modulus of elasticity, D_p the internal diameter of the pipe, h the pipe wall thickness, ν the Poisson ratio. By substituting equation (6) into equation (10b), the following equation can be derived:

$$\dot{A} + UA' - \frac{2A\sqrt{A}}{\sqrt{\pi Eh}} (\dot{p} + Up') + \frac{\nu\sqrt{AA_t}}{\sqrt{\pi h}} (\dot{u}' + w'\dot{w}' + U(u'' + w'w'')) = 0. \quad (11)$$

Upon substituting equations (6) and (10) into equation (9), the continuity equation of unsteady flow can also be expressed as

$$\dot{p} + Up' + \rho_f a^2 (U' - 2v\dot{u}') - 2v\rho_f a^2 (w'\dot{w}' + Uu'' + Uw'w'') = 0, \quad (12)$$

where a is the fluid wave speed, defined through

$$a^2 = \frac{K/\rho_f}{1 + KD_p/(Eh)}. \quad (13)$$

When the vibration of the pipe is not considered, equation (12) is equivalent to the equation used by Wylie and Streeter [13]. The tangential force f_t per unit length acting on the pipe walls can be expressed in the form [13]

$$f_t = \tau_0 \pi D_p = \frac{\pi}{8} D_p \rho_f f_s |U|U, \quad (14)$$

where f_s is the Darcy–Weisbach friction factor [14], and τ_0 is the shear stress on fluid–pipe interface. By eliminating the term f_n from equations (1) and (2), and using equation (14), the following momentum equation of unsteady flow can be obtained:

$$\begin{aligned} (pA)' + m_f(\dot{U} + UU' + f_s|U|U/2D_p + g \sin \phi + gw' \cos \phi) \\ + m_f(\ddot{u} + 2U\dot{u}' + \dot{U}u' + U^2u'') + m_f(\ddot{w} + 2U\dot{w}' + \dot{U}w' \\ + UU'w' + U^2w'' + \ddot{w}_0)w' = 0. \end{aligned} \quad (15)$$

The equations of waterhammer, equations (12) and (15), are fully coupled with the vibration of the fluid-conveying pipe in the lateral and longitudinal directions. When the pipe conveying fluid does not vibrate in either direction, equations (12) and (15) become identical to the equations of continuity and momentum in the classical waterhammer theory used by Wylie and Streeter [13].

Five non-linear partial differential equations (7), (8), (11), (12) and (15) governing the motion of the fluid and pipe have been derived, and are fully coupled through the equilibrium of contact forces and the normal compatibility of velocity in the fluid–pipe interfaces, and conservation of mass and momentum of the transient fluid. From equations (11) and (12), it can also be seen that the Poisson coupling and the dilation due to the change of hydrodynamic pressure are considered. If the non-linear terms associated with displacements, \ddot{w}_0 , T_0 and c in equation (7), the second to fourth and fifth to seventh terms in equation (8), the fourth term in equation (12) and the third and fourth terms in equation (15) are neglected, these equations become identical to the analytical model used by Lee *et al.* [11].

Boundary conditions to complement the equations of motion (7) and (8) for a pipe simply supported at both ends are

$$\begin{aligned} w(0, t) = w''(0, t) = 0, \quad w(L, t) = w''(L, t) = 0, \\ u(0, t) = u''(0, t) = 0, \quad u(L, t) = u''(L, t) = 0. \end{aligned} \quad (16)$$

together with the boundary conditions

$$\begin{aligned} \bar{w}(0, \bar{t}) = \bar{w}''(0, \bar{t}) = 0, & \quad \bar{w}(1, \bar{t}) = \bar{w}''(1, \bar{t}) = 0, \\ \bar{u}(0, \bar{t}) = \bar{u}''(0, \bar{t}) = 0, & \quad \bar{u}(1, \bar{t}) = \bar{u}''(1, \bar{t}) = 0. \end{aligned} \tag{23}$$

3. NUMERICAL SOLUTION

A finite difference method (FDM) is applied directly to solve the partial differential equations (18)–(20) for the dynamic response of the fluid–pipe system. The fluid–pipe system is discretized into n short segments. Terms in these equations involving only the time derivatives with even order or spatial derivatives are approximated by central differences, namely,

$$\begin{aligned} \left(\frac{\partial^2 \bar{w}}{\partial \bar{t}^2}\right)_j^i &= \frac{\bar{w}_j^{i+1} - 2\bar{w}_j^i + \bar{w}_j^{i-1}}{\Delta \bar{t}^2}, & \left(\frac{\partial^2 \bar{w}}{\partial \bar{x}^2}\right)_j^i &= \frac{\bar{w}_{j+1}^i - 2\bar{w}_j^i + \bar{w}_{j-1}^i}{\Delta \bar{x}^2}, \\ \left(\frac{\partial^2 \bar{u}}{\partial \bar{t}^2}\right)_n^i &= \frac{\bar{u}_j^{i+1} - 2\bar{u}_j^i + \bar{u}_j^{i-1}}{\Delta \bar{t}^2}, & \left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}\right)_j^i &= \frac{\bar{u}_{j+1}^i - 2\bar{u}_j^i + \bar{u}_{j-1}^i}{\Delta \bar{x}^2}, \\ \left(\frac{\partial^4 \bar{w}}{\partial \bar{x}^4}\right)_j^i &= \frac{\bar{w}_{j+2}^i - 4\bar{w}_{j+1}^i + 6\bar{w}_j^i - 4\bar{w}_{j-1}^i + \bar{w}_{j-2}^i}{\Delta \bar{x}^4}, & \left(\frac{\partial \bar{w}}{\partial \bar{x}}\right)_j^i &= \frac{\bar{w}_{j+1}^i - \bar{w}_{j-1}^i}{2\Delta \bar{x}}, \\ \left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)_j^i &= \frac{\bar{u}_{j+1}^i - \bar{u}_{j-1}^i}{2\Delta \bar{x}}, & \left(\frac{\partial \bar{p}}{\partial \bar{x}}\right)_j^i &= \frac{\bar{p}_{j+1}^i - \bar{p}_{j-1}^i}{2\Delta \bar{x}}, & \left(\frac{\partial \bar{U}}{\partial \bar{x}}\right)_j^i &= \frac{\bar{U}_{j+1}^i - \bar{U}_{j-1}^i}{2\Delta \bar{x}}, \\ \left(\frac{\partial \bar{A}}{\partial \bar{x}}\right)_j^i &= \frac{\bar{A}_{j+1}^i - \bar{A}_{j-1}^i}{2\Delta \bar{x}}, \end{aligned} \tag{24}$$

where the superscript i indicates the time step, $i = 1, 2, \dots, s$ and s is the total number of time steps; the subscript j indicates the spatial node, $j = 1, 2, \dots, n + 1$ and $(n + 1)$ is the total number of spatial nodes; the dimensionless segment length is $\Delta \bar{x} = 1/n$. For simplicity, forward differences with respect to time are used for terms associated with pressure, velocity and area in equations (18) and (19). Backward differences with respect to time are used for the terms associated with displacements of first order in equations (18)–(22), i.e.,

$$\begin{aligned} \left(\frac{\partial \bar{p}}{\partial \bar{t}}\right)_j^i &= \frac{\bar{p}_j^{i+1} - \bar{p}_j^i}{\Delta \bar{t}}, & \left(\frac{\partial \bar{U}}{\partial \bar{t}}\right)_j^i &= \frac{\bar{U}_j^{i+1} - \bar{U}_j^i}{\Delta \bar{t}}, & \left(\frac{\partial \bar{A}}{\partial \bar{t}}\right)_j^i &= \frac{\bar{A}_j^{i+1} - \bar{A}_j^i}{\Delta \bar{t}}, \\ \left(\frac{\partial \bar{w}}{\partial \bar{t}}\right)_j^i &= \frac{\bar{w}_j^i - \bar{w}_j^{i-1}}{\Delta \bar{t}}, & \left(\frac{\partial^2 \bar{w}}{\partial \bar{x} \partial \bar{t}}\right)_j^i &= \frac{(\bar{w}_{j+1}^i - \bar{w}_{j-1}^i) - (\bar{w}_{j+1}^{i-1} - \bar{w}_{j-1}^{i-1})}{2\Delta \bar{x} \Delta \bar{t}}, \\ \left(\frac{\partial^2 \bar{u}}{\partial \bar{x} \partial \bar{t}}\right)_j^i &= \frac{(\bar{u}_{j+1}^i - \bar{u}_{j-1}^i) - (\bar{u}_{j+1}^{i-1} - \bar{u}_{j-1}^{i-1})}{2\Delta \bar{x} \Delta \bar{t}}. \end{aligned} \tag{25}$$

The method of characteristics (MOC) is applied to the partial differential equations (21) and (22), and the partial differential terms associated with flow velocity and hydrodynamic pressure can be rewritten as ordinary differential ones compatible with two characteristic lines. The partial differential terms associated with displacements in equations (21) and (22) are treated by a FDM. The following two sets of equations of the characteristics (C^+ and C^-) can be obtained:

$$C^+: \left\{ \begin{array}{l} \frac{d\bar{U}}{d\bar{t}} + \frac{1}{\bar{a}} \frac{d\bar{p}}{d\bar{t}} + \frac{\chi}{\sqrt{\beta\bar{A}}} |\bar{U}| \bar{U} - 2v\bar{a} \left(\dot{u}' + \bar{w}' \dot{w}' + \frac{1}{\sqrt{\beta}} \bar{U} \bar{u}'' + \frac{1}{\sqrt{\beta}} \bar{U} \bar{w}' \bar{w}'' \right) \\ \quad + (1/\sqrt{\beta}) \bar{p} \bar{A}' + (\bar{g}/\sqrt{\beta}) (\sin \phi + \bar{w}' \cos \phi) = 0, \\ \frac{d\bar{x}}{d\bar{t}} = \frac{1}{\sqrt{\beta}} (\bar{U} + \bar{a}), \end{array} \right. \quad (26)$$

$$C^-: \left\{ \begin{array}{l} \frac{d\bar{U}}{d\bar{t}} - \frac{1}{\bar{a}} \frac{d\bar{p}}{d\bar{t}} + \frac{\chi}{\sqrt{\beta\bar{A}}} |\bar{U}| \bar{U} + 2v\bar{a} \left(\dot{u}' + \bar{w}' \dot{w}' + \frac{1}{\sqrt{\beta}} \bar{U} \bar{u}'' + \frac{1}{\sqrt{\beta}} \bar{U} \bar{w}' \bar{w}'' \right) \\ \quad + (1/\sqrt{\beta}) \bar{p} \bar{A}' + (\bar{g}/\sqrt{\beta}) (\sin \phi + \bar{w}' \cos \phi) = 0, \\ \frac{d\bar{x}}{d\bar{t}} = \frac{1}{\sqrt{\beta}} (\bar{U} - \bar{a}), \end{array} \right. \quad (27)$$

As the combination of the FDM and the MOC is employed for the analysis of dynamic response of a pipeline conveying fluid with unsteady flow, $\Delta\bar{t}/\Delta\bar{x}^2 = 0.01 \sqrt{EI/m}$ and $\Delta\bar{t}/\Delta\bar{x} = \sqrt{\beta}/(\bar{U} + \bar{a})$ are chosen for the FDM and MOC respectively during computation to ensure numerical stability. In this analysis, $\bar{a} \gg \bar{U}$ and the influence of the change of cross-sectional area \bar{A} on the wave speed is neglected. During the transient response of the system, necessary boundary and initial conditions are imposed on equations (26) and (27); flow velocity and hydrodynamic pressure can then be solved. With known velocities and pressures, it is possible to solve equations (18)–(20) to determine the internal cross-sectional area and displacements of the pipe. The displacements and cross-sectional area of the pipe will now permit the solution of equations (26) and (27) again using the boundary conditions and previous parameters. This iterative procedure may be repeated for the necessary time period.

4. A NUMERICAL EXAMPLE

A program was written to compute the dynamic response of a pipeline conveying fluid with a pulsating flow, and to examine the effect of the pressure wave on the vibration and *vice versa*. For the numerical calculation, the pipeline system used by Lee *et al.* [11] was followed. The pipeline is inclined and simply supported at both ends. The pulsating flow velocity, $U = U_0(1 + \mu \cos \omega t)$, is obtained by the

operation of a valve located downstream. At the entry section, water head is assumed to be constant. The following dimensionless variables are used:

$$\bar{D} = D_p/L, \quad \Omega = \omega L^2 \sqrt{m/EI}, \quad (28)$$

where ω is the valve frequency, U_0 is the constant mean flow velocity, and μ is the small excitation parameter. The pipeline conveying fluid has the following characteristics: $\bar{U}_0 = 1.0$, $\bar{g} = 0.05$, $\bar{p}_0 = 1.0$, $\phi = 0.3$, $f_s = 0.02$, $\beta = 0.3$, $\bar{D} = 0.05$, $\mu = 0.15$. As lateral, longitudinal and radial vibration coupling and waterhammer have been considered in this analysis, more dimensionless parameters to complement the closed-form solution of equations (18)–(20), (26) and (27) are required as follows: $h/D_p = 0.05$, $\varphi_1 = \bar{c} = 0$, $\alpha = 2.1$, $\nu = 0.35$, $K/E = 3.738$.

To initiate vibrations, initial static distributed displacements, which start from the equilibrium position of the pipeline conveying fluid, were imposed. The pipeline was discretized into 20 short segments. The computed hydrodynamic pressures of the fluid and displacements of the pipe are compared with the existing results [11] and results obtained by using Lee's model [11], outlined in Appendix A.

Figure 2 shows the comparison of results obtained by using our model with existing results [11]. The pressure fluctuation computed by using our model is much larger than the ones obtained by neglecting the effect of waterhammer. Despite the large difference in pressure, lateral displacements obtained by our model are a little larger than those in reference [11].

Figures 3 and 4 show the effect of the pulsation frequency, Ω , on the dynamic response of the pipeline conveying fluid at $\bar{x} = 0.25$. The comparison of hydrodynamic pressure, displacement and flow velocity histories obtained by using our model with those obtained by using Lee's model [11] has been made for $\Omega = 5$ and 1. For these cases, lateral displacements remain very close, but there is a much bigger difference in the frequency of lateral vibration between the two methods, due to the waterhammer wave and the lateral, longitudinal and radial vibration coupling as well.

It can be seen from Figures 2–4 that the transient hydrodynamic pressure wave and vibration coupling trigger the vibration of the pipeline conveying fluid with a higher frequency. As the pulsating frequency decreases, hydrodynamic pressures and longitudinal displacements obtained by our model decrease dramatically.

Figures 3 and 4 also show that the hydrodynamic pressures are composed of two periodic components, one due to the pulsating flow and the other due to the water hammer waves. It is observed in Figure 4 that the waterhammer frequency is 15 times the pulsation frequency. Decreasing the pulsation frequency, the flow velocities at $\bar{x} = 0.25$ draw gradually close to the flow velocities at the outflow section.

5. CONCLUSIONS

The five non-linear equations governing the motion of a pipeline conveying fluid with unsteady flow have been derived. These fully coupled partial differential equations can be solved by the combination of a finite difference method and

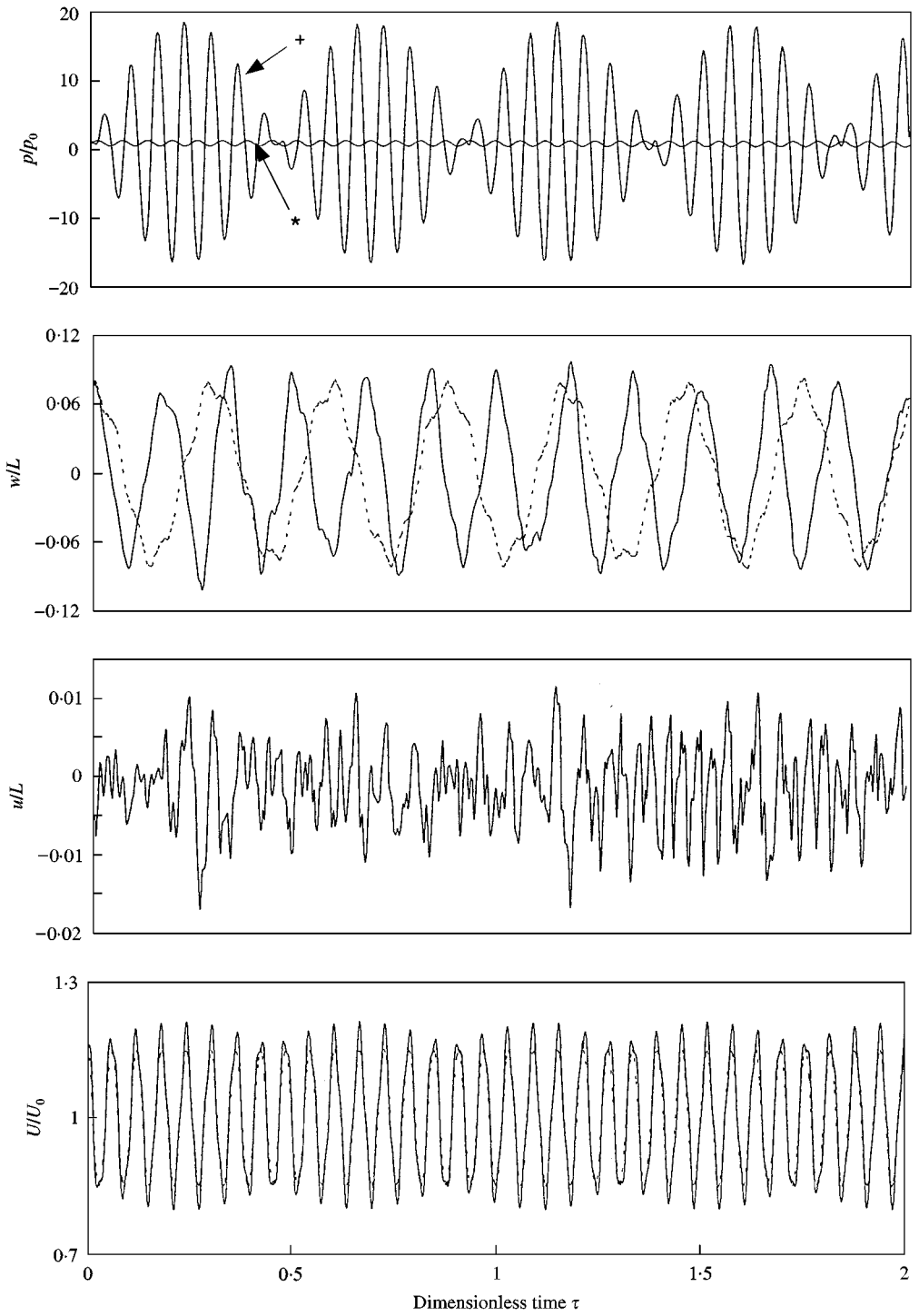


Figure 2. Dynamic response of pipe conveying fluid at $\bar{x} = 0.25$ for $\Omega = 16.5$. +, — Present model; *, ---- reference [11].

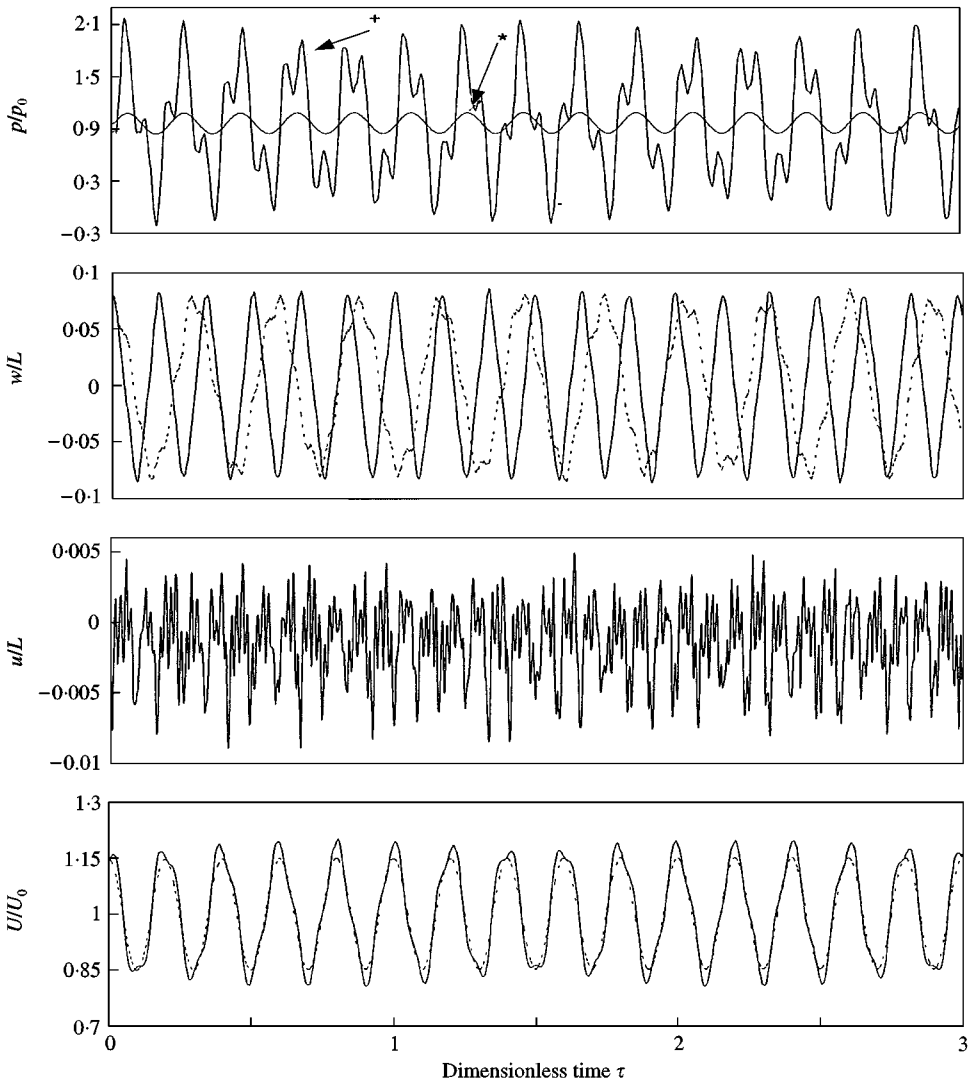


Figure 3. Dynamic response of pipe conveying fluid at $\bar{x} = 0.25$ for $\Omega = 5$. +, — Present model; *, ---- Lee's model [11].

a method of characteristics to predict more accurately the dynamic response of the system, and the hydrodynamic pressures and flow velocities of the fluid. Poisson coupling, the equilibrium of fluid–pipe interface forces and the normal compatibility of fluid–pipe interface velocities have been considered in our model. The displacement, hydrodynamic pressures and flow velocities obtained by our model for a simply supported pipe conveying a sinusoidal flow were compared with existing theoretical results and results obtained by using another model [11]. Transient hydrodynamic pressure waves trigger a vibration of the pipeline with a higher frequency. With increasing pulsation frequencies, longitudinal vibration tends to be larger. The magnitudes of hydrodynamic pressures are larger than those

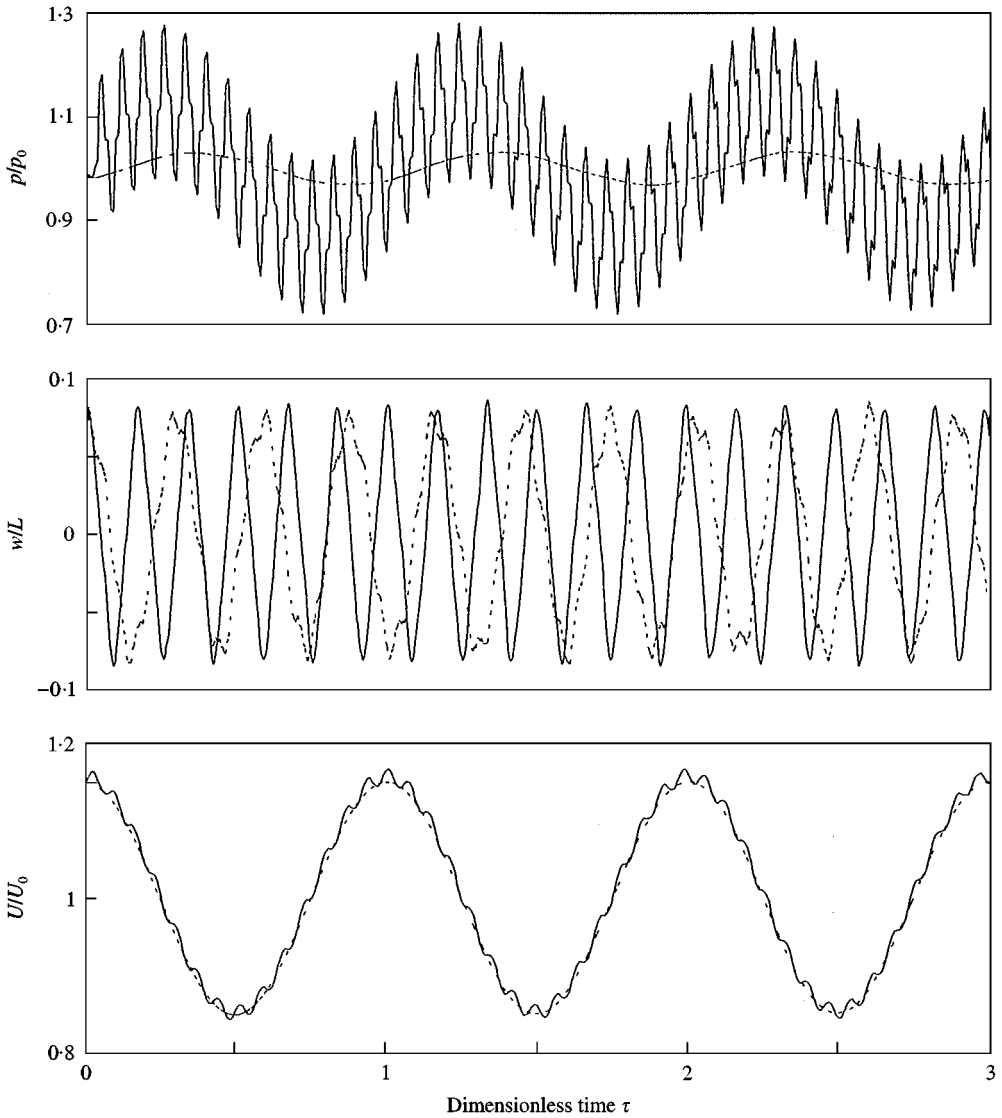


Figure 4. Dynamic response of pipe conveying fluid at $\bar{x} = 0.25$ for $\Omega = 1$. +, — Present model; *, ---- Lee's model [11].

which were obtained without full consideration of the fluid–structure interaction. Although these results appear promising, they must be confirmed experimentally. The experimental set up and comparison with numerical results will be the subject of a future paper.

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APPENDIX A: LEE'S MODEL

The axial and transverse vibrational equations of a pipeline and the fluid continuity and momentum equations derived by Lee *et al.* [11] are given by

$$m_t \ddot{u} - EA_t u'' + m_f \dot{U} + m_f U U' + (pA)' - mg \sin \phi = 0. \quad (\text{A1})$$

$$\begin{aligned} m \ddot{w} + 2m_f U \dot{w}' + m_f \dot{U} w' + m_f U U' w' + (pA)' w' + m_f U^2 w'' + (pA) w'' \\ + EI w'''' - mg \cos \phi = 0, \end{aligned} \quad (\text{A2})$$

$$\dot{p} + U p' + \rho_f a^2 (U' - 2v \dot{u}') = 0, \quad (\text{A3})$$

$$(pA)' + m_f (\dot{U} + U U' + f_s |U| U / 2D_p + g \sin \phi + g w' \cos \phi) = 0. \quad (\text{A4})$$

Numerical calculation of the dynamic response of a pipeline for several flow pulsation frequencies was performed by Lee *et al.* using the following dimensionless

equations which were obtained after neglecting small coupling terms and assuming $u = 0$ and $\bar{U}' = 0$ [11]:

$$\ddot{w} + 2\sqrt{\beta}\bar{U}\dot{w}' + \sqrt{\beta}\dot{\bar{U}}\bar{w}' + \bar{p}'\bar{w}' + \bar{U}^2\bar{w}'' + \bar{p}\bar{w}'' + \bar{w}'''' + \bar{g}\cos\phi = 0, \quad (\text{A5})$$

$$\sqrt{\beta}\dot{\bar{U}} + \chi|\bar{U}|\bar{U} + \bar{p}' + \bar{g}(\sin\phi + \bar{w}'\cos\phi) = 0. \quad (\text{A6})$$

Equations (A5) and (A6) together with the assumption were called Lee's simplified model of equations (A1)–(A4), and solved by using a central difference method.